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Experimental measurements of coating mechanical loss factors

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Abstract
All current gravitational wave detectors use test masses coated with alternating layers of two different dielectric materials to form highly reflective mirrors. The thermal noise from mechanical dissipation associated with such coatings may be significant for future detectors such as advanced LIGO. We have measured the mechanical dissipation of a number of types of coatings formed from SiO_2 (silica) and Ta_2O_5 (tantala). The frequency dependence of the dissipation has been determined, taking into account the contribution of thermoelastic loss.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Interferometric gravitational wave detectors operate by sensing small changes in the relative displacements of suspended test masses that have been coated to form mirrors. An important limit to the sensitivity of such detectors in the frequency region from approximately ten Hz to a few hundred Hz is set by the thermal noise contributed by these test masses. For the proposed advanced LIGO detectors, the most promising candidates for the test mass material are fused silica and sapphire [1]. The addition of a dielectric coating to a test mass substrate will lead to an increase in the thermal noise of the test mass sensed in a laser interferometer over that resulting from the mechanical dissipation from the bulk material of the test mass

[2–4]. Experimental measurements of the mechanical dissipation of silica substrates with ion-beam-sputtered dielectric coatings have shown that the mechanical loss of such a coating is in the range 10^{-4} to 10^{-3}. An estimate of the thermal noise level calculated from such values suggests that this could pose a problem for proposed room temperature advanced detectors [5–7], although operation at cryogenic temperatures should reduce the level of thermal noise resulting from coating mechanical dissipation [8]. Recent work has shown that thermoelastic damping is a frequency dependent dissipation mechanism which needs to be considered both in the interpretation of the measured mechanical losses [9] and also in the calculation of expected thermal noise levels [9, 10]. Moreover, since the experimental measurements were made at frequencies from approximately 2–74 kHz (well above the frequency band of interest for ground based detectors) it is important to examine any frequency dependence of the mechanical dissipation of the coatings.

2. Coating losses

In general, the power spectral density of the thermally induced displacement of any system is given by [2]

$$|x^2(f)| = \frac{4k_B T}{\pi f} U_{\mathrm{stored}} \phi(f), \tag{1}$$

where T is the temperature, f is the frequency, U_{stored} is the energy stored in the system and $\phi(f)$ is the loss of the system. In this context, we are particularly interested in the loss attributable to the coating. If the intrinsic loss of the material of the substrate $\phi(f_0)_{\mathrm{s}}$ and the loss of the coating $\phi(f_0)_{\mathrm{c}}$ are the only losses contributing to the measured loss of a coated sample at a resonant frequency f_0, then this loss can be described in the following way:

$$\phi(f_0)_{\mathrm{cs}} = \phi(f_0)_{\mathrm{s}} + \frac{U_{\mathrm{c}}}{U_{\mathrm{s}}} \phi(f_0)_{\mathrm{c}}, \tag{2}$$

where $\phi(f_0)_{\mathrm{cs}}$ is the loss of the coated sample, $U_{\mathrm{c}}/U_{\mathrm{s}}$ is the ratio of the strain energy stored in the coating to that stored in the substrate and is determined using finite element analysis. $\phi(f_0)_{\mathrm{s}}$ and $\phi(f_0)_{\mathrm{cs}}$ are determined experimentally [7]. Recent work [9] has shown that part of the coating loss $\phi(f_0)_{\mathrm{c}}$ is attributable to thermoelastic damping. This type of damping arises from the coating and substrate having different thermal properties. We may express the total coating loss as $\phi_{\mathrm{c}}(f) = \phi_{\mathrm{th}}(f) + \phi_{\mathrm{residual}}(f)$ where $\phi_{\mathrm{residual}}(f)$ is the residual loss, which is potentially due to the intrinsic mechanical properties of the coating [7], and $\phi_{\mathrm{th}}(f)$, the thermoelastic term, is given by

$$\phi_{\mathrm{th}}(f) = \frac{2C_F T}{\left(\frac{E}{1-\nu}\right)_{\mathrm{avg}}} \left[\frac{1}{C_F} \left(\frac{E\alpha}{1-\nu} \right)_{\mathrm{avg}} - \frac{1}{C_s} \frac{E_s \alpha_s}{1-\nu_s} \right]^2 g(f), \tag{3}$$

where the frequency dependence term $g(f)$ is given by

$$g(f) = \mathrm{Im}\left[-\frac{1}{\sqrt{\mathrm{i}2\pi f \tau_F}} \frac{\sinh\sqrt{\mathrm{i}2\pi f \tau_F}}{\cosh\sqrt{\mathrm{i}2\pi f \tau_F} + \sqrt{k_F C_F / k_s C_s}\,\sinh\sqrt{\mathrm{i}2\pi f \tau_F}} \right], \tag{4}$$

with $\tau_F = l^2 C_F / k_F$ and where E is Young's modulus, C is heat capacity per unit volume, α is coefficient of thermal expansion, l is coating thickness, k is thermal conductivity and ν is Poisson's ratio. $k_F^{-1} \equiv (k^{-1})_{\mathrm{avg}}$ and $C_F \equiv (C)_{\mathrm{avg}}$. In equations (3) and (4) s denotes a substrate property and avg denotes properties being averaged in proportion to their fractional amounts

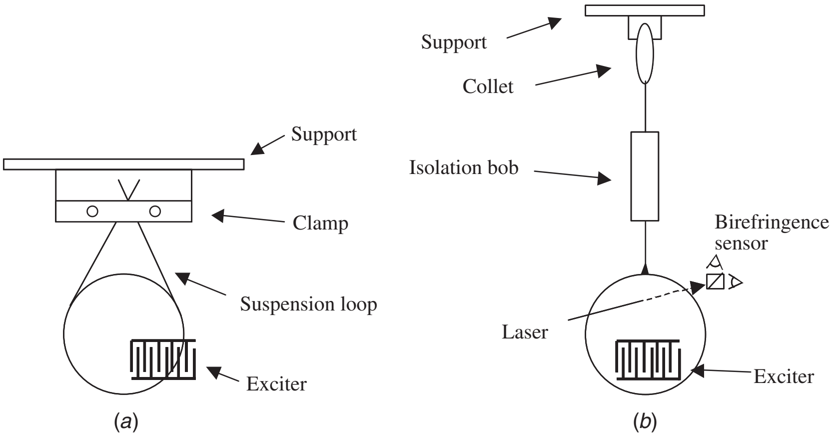

Figure 1. Shown are the experimental arrangements used in the measurements of the loss factors of coated silica substrates for samples of thickness (*a*) 2.54 cm and (*b*) 0.25 cm.

Table 1. Dielectric coatings studied in this investigation. Here $\lambda = 1064$ nm.

Coating number	Total layers	First material	Optical thickness per layer	Second material	Optical thickness per layer
1	30	Silica	$\lambda/4$	Tantala	$\lambda/4$
2	30	Silica	$3\lambda/8$	Tantala	$\lambda/8$
3	30	Silica	$\lambda/8$	Tantala	$3\lambda/8$

in a coating. For a coating consisting of two alternating materials a and b of thicknesses t_a and t_b, respectively, we define the volume averaging operator avg by

$$(X)_{\mathrm{avg}} \equiv \frac{t_a}{t_a + t_b} X_a + \frac{t_b}{t_a + t_b} X_b. \tag{5}$$

3. Experimental details

The loss factors of a number of coated silica substrates were measured. Substrates with two different aspect ratios were used; one with a diameter of 7.62 cm and a thickness of 2.54 cm and another with diameter 7.62 cm and thickness 0.25 cm. In each case a number of resonant modes of the mass were excited and the subsequent ringdown recorded to calculate loss factors in each case. The measured resonant modes of the thin samples lay in the region 2–6 kHz whilst those of the thick samples lay in the region 20–74 kHz.

The experimental setups for the measurements are those presented in [7]. The suspension techniques used are shown schematically in figure 1.

The use of samples of two aspect ratios allowed measurements to be made over a wide range of frequencies. The coatings studied here are given in table 1. For each aspect ratio, two samples were typically measured.

4. Results and analysis

Our previous studies have suggested [7] that the source of dissipation in dielectric coatings is due to the coating materials themselves and not friction between the coating layers or

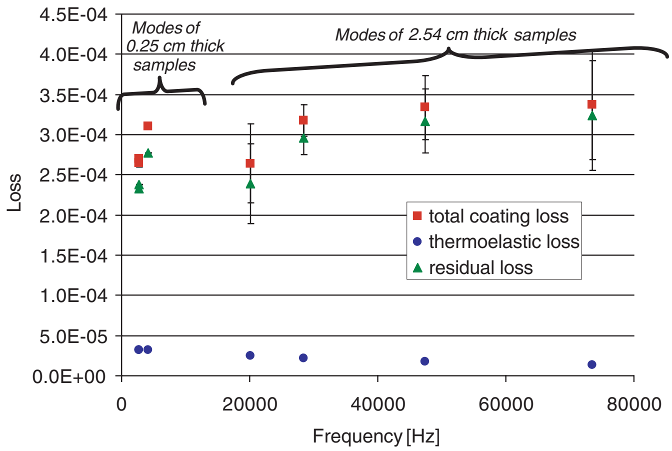

Figure 2. Graph showing experimental coating losses, calculated thermoelastic coating losses and residual coating losses as a function of frequency for coating 1.

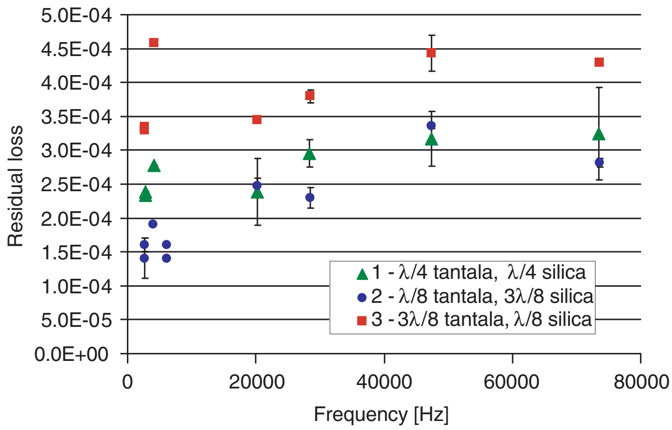

Figure 3. Comparison of residual losses as a function of frequency for 30-layer coatings containing different relative levels of silica and tantala.

between the coating and the substrate. It is important to analyse how the measured loss is split between damping due to thermoelastic effects and due to other internal dissipation effects in the coating materials. The total coating loss, obtained using our experimental measurements and equation (2), as well as the calculated thermoelastic loss $\phi_{\mathrm{th}}(f)$ and residual coating loss $\phi_{\mathrm{residual}}(f)$ are shown for coating 1 in figure 2.

From figure 2 it can be seen that for this particular coating/substrate combination, thermoelastic effects are small compared to other internal dissipation effects. However, this will not necessarily be the case for other coating/substrate combinations.

4.1. Composition study

It has previously been shown [7] that the total level of dissipation associated with a silica/tantala coating depends on the relative amounts of silica and tantala in the coating. Figure 3 shows

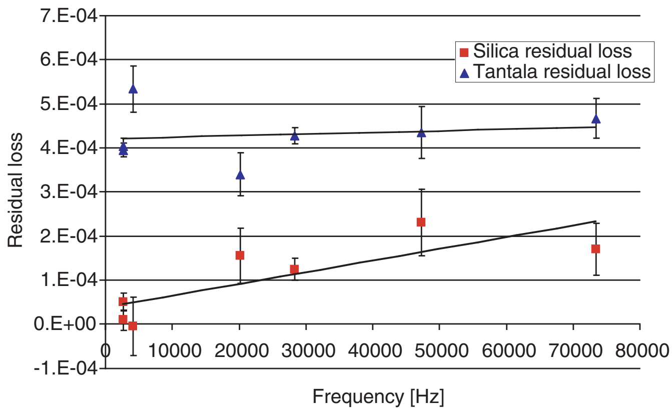

Figure 4. Residual losses for silica, ϕ_{Sc} and tantala, ϕ_{Tc}, as a function of frequency.

the residual losses for coatings 1–3 plotted as a function of frequency, where each coating contains different amounts of silica and tantala.

It can be seen from this figure that the magnitude of the residual loss depends on the relative amounts of each coating material present. For any mode, the contribution to the overall coating loss of each coating material is obtainable using [7]

$$\phi_{\mathrm{residual}}(f) = \phi_{\mathrm{Sc}}(f)A + \phi_{\mathrm{Tc}}(f)B, \tag{6}$$

where

$$A = \left[\frac{E_{\mathrm{Sc}}t_{\mathrm{Sc}}}{E_{\mathrm{Sc}}t_{\mathrm{Sc}} + E_{\mathrm{Tc}}t_{\mathrm{Tc}}}\right] \qquad B = \left[\frac{E_{\mathrm{Tc}}t_{\mathrm{Tc}}}{E_{\mathrm{Sc}}t_{\mathrm{Sc}} + E_{\mathrm{Tc}}t_{\mathrm{Tc}}}\right], \tag{7}$$

and $\phi_{\mathrm{Sc}}(f)$ is the loss of the silica and $\phi_{\mathrm{Tc}}(f)$ is the loss of the tantala in the coating. E and t are the Young's modulus and thickness of each material, respectively. Note that this equation neglects the effects of Poisson's ratio; these are, however, expected to be small. For each mode studied, A and B were calculated for the different coatings and then multiple regression was used to calculate $\phi_{\mathrm{Sc}}(f)$ and $\phi_{\mathrm{Tc}}(f)$. The results of this analysis are shown in figure 4.

For simplicity, the loss of each material can be modelled as having a linear dependence on frequency such that $\phi_{\mathrm{material}}(f) = \phi_0 + f\phi_{\mathrm{f}}$. The results for each material are

$$\phi_{\mathrm{Sc}}(f) = (0.4 \pm 0.3) \times 10^{-4} + f(2.7 \pm 0.9) \times 10^{-9}, \tag{8}$$

$$\phi_{\mathrm{Tc}}(f) = (4.2 \pm 0.4) \times 10^{-4} + f(0.4 \pm 0.9) \times 10^{-9}. \tag{9}$$

This indicates that the magnitude of the residual loss of the tantala component of a coating is larger than that of the silica component, but is largely frequency independent. The residual loss of the silica component is lower than that of the tantala component and appears to decrease towards lower frequencies. Note that subtracting the calculated thermoelastic loss has resulted in individual values for the residual loss of the silica component of the coating which are zero or negative at low frequencies. However, little emphasis should be placed on this because of the size of the estimated errors. This may be a result of uncertainty in the parameters used to calculate the thermoelastic loss resulting in an overestimation of the loss. In addition, it should be noted that thermoelastic loss is not associated with shear deformation of a coating [9]. Thus, the level of thermoelastic loss associated with a given mode of a sample may be lower than simply estimated here by a factor dependent on the fractional energy associated with shear deformation of the coating. This effect is currently being studied. Having determined the

Table 2. Parameters used in calculation of thermoelastic noise level [9, 11–16].

Property	Value	Property	Value
n_{silica}	1.45	α_{silica}	5.1×10^{-7} K^{-1}
n_{tantala}	2.03	α_{tantala}	3.6×10^{-6} K^{-1}
E_{silica}	7.2×10^{10} Pa	C_{silica}	1.64×10^{6} JK^{-1} m^{-3}
E_{tantala}	1.4×10^{11} Pa	C_{tantala}	2.51×10^{6} JK^{-1} m^{-3}
ν_{silica}	0.17	k_{silica}	1.38 Wm^{-1} K^{-1}
ν_{tantala}	0.23	k_{tantala}	33 Wm^{-1} K^{-1}

Table 3. Shown are the contributions to the total coating thermal noise from (a) the residual coating loss and (b) thermoelastic dissipation for a single test mass. Note that the individual noise values are added in quadrature to give the total.

Type of thermal noise	Noise level (m Hz$^{-1/2}$) at 100 Hz
(a) Residual	9.7×10^{-21}
(b) Thermoelastic	2.1×10^{-21}
Total	9.9×10^{-21}
AdvLIGO target	6×10^{-21}[18]

relative levels of loss attributable to each material in the coating, it is possible to calculate the level of thermal noise expected from a silica/tantala coating typical of the type planned for use in advanced gravitational wave detectors.

5. Thermal noise calculation

When calculating the expected thermal noise associated with the dielectric mirror coatings in a gravitational wave detector, it is important to include both the thermoelastic coating thermal noise as well as the thermal noise due to the residual coating loss. The expression for the power spectral density of the thermoelastic thermal noise from a coating on a substrate is given by [9]

$$S_x(f) = \frac{8k_B T^2}{\pi^2 f} \frac{1}{w^2} \frac{\alpha_s^2 C_F}{C_s^2} (1+\nu_s)^2 \tilde{\Delta}^2 g(f), \tag{10}$$

where

$$\tilde{\Delta}^2 = \left\{ \frac{C_s}{2\alpha_s C_F} \left(\frac{\alpha}{1-\nu} \left[\frac{1+\nu}{1+\nu_s} + \left(1 - 2\nu_s \frac{E}{E_s} \right) \right] \right)_{\text{avg}} - 1 \right\}^2 \tag{11}$$

and w is beam radius (the distance for the amplitude of a laser beam to drop to $1/e$ of its original value). The thermoelastic coating noise and residual coating thermal noise were calculated for a silica/tantala coating with a 15 ppm transmission applied to a single fused silica test mass. This transmission is typical of that which may be used in future detectors. The results of this calculation are shown in table 3. The parameters used in the thermoelastic coating noise are given in table 2.

It should be noted that the thermal noise contribution of the coating alone is greater than the target thermal noise level for advanced LIGO at 100 Hz. However, this noise is primarily due to the residual coating loss, as opposed to that due to the thermoelastic loss. Furthermore, it may be recalled that the residual loss is dominated by that due to tantala. Hence, one possible way forward would be to search for a method of reducing the residual loss of tantala. Another would be to look for a material with similar thermo-mechanical properties to tantala, but with a lower residual loss.

6. Conclusions

Both thermoelastic noise and noise resulting from residual dissipation in the coating materials are of a level which is significant when calculating coating thermal noise. The results presented here suggest that a promising route to reducing coating thermal noise may be through reducing the residual mechanical loss of tantala or alternatively by choosing an alternate high-index material with similar thermo-mechanical properties but lower residual mechanical loss. Experiments suggest that the addition of a dopant to the tantala in the coating may reduce the mechanical dissipation [17].

An area requiring further study is the determination of accurate values for the thermo-mechanical properties of the ion-beam-sputtered coating materials as deposited. This will help improve the accuracy of the calculation of levels of thermoelastic noise associated with coatings. In addition, further measurements of coating mechanical losses at frequencies closer to the bandwidth of interest of ground-based gravitational wave detectors are desirable. It should also be noted that optical coating properties must be studied in parallel with thermo-mechanical coating properties.

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References

[1] Gustafson E, Shoemaker D, Strain K and Weiss R 1999 LSC White paper on detector research and development LIGO T990080-00-D
[2] Levin Y 1998 *Phys. Rev.* D **57** 659
[3] Yamamoto K 2000 Study of the thermal noise caused by inhomogeneously distributed loss *PhD Thesis* University of Tokyo
[4] Nakagawa N, Gretarsson A M, Gustafson E K and Fejer M M 2002 *Phys. Rev.* D **65** 102001
[5] Crooks D R M *et al* 2002 *Class. Quantum Grav.* **19** 883–96
[6] Harry G *et al* 2002 *Class. Quantum Grav.* **19** 897–917
[7] Penn S D *et al* 2003 *Class. Quantum Grav.* **20** 2917–28
[8] Yamamoto K 2003 Mechanical loss of optical coating at low temperature LIGO G030225-00-Z
[9] Fejer M M, Rowan S, Cagnoli G, Crooks D R M, Gretarsson A, Harry G M, Hough J, Penn S D, Sneddon P H and Vyatchanin S P *Phys. Rev.* D submitted
[10] Braginsky V B and Samoilenko S S 2003 *Phys. Lett.* A **315** 175–7
[11] REO Inc., 5505 Airport Blvd Boulder, Colorado 80301 USA, private communication
[12] Waynant R W and Ediger M N 1994 *Electro-Optics Handbook* (New York: McGraw-Hill)
[13] Musikant S 1985 *Optical Materials* (New York: Dekker)
[14] Tien C L, Jaing C C, Lee C C and Chuang K P 2000 *J. Mod. Opt.* **47** 1681
[15] Martin P J, Bendavid A, Swain M V, Netterfield R P, Kinder T J, Sainty W G and Drage D 1993 *Proc. Symp. on Thin Films: Stresses and Mechanical Properties IV*
[16] Samsonov G V 1982 *The Oxide Handbook* 2nd edn (New York: IFI/Plenum)
[17] Harry G 2003 Thermal noise from optical coatings LIGO G030337-00
[18] Fritschel P (ed) 2001 Advanced LIGO systems design LIGO-T010075-00-D