# Tandem chirped quasi-phase-matching grating optical parametric amplifier design for simultaneous group delay and gain control 

M. Charbonneau-Lefort and M. M. Fejer<br>E. L. Ginzton Laboratory, Stanford University, Stanford, California 94305<br>Bedros Afeyan<br>Polymath Research Inc., Pleasanton, California 94566

Received June 21, 2004


#### Abstract

We present a broadband optical parametric amplifier design using tapered gain and tandem chirped quasi-phase-matching gratings to obtain flat gain and group-delay spectra suitable for applications such as ultrashort-pulse amplification and fiber-optic communication systems. Although a tapered-gain amplifier consisting of a single chirped grating can provide constant gain over a wide frequency range, it cannot be used to control the group delay across the spectrum. We propose controlling both the gain and the group delay profiles using a two-stage amplifier configuration, in which the idler of the first is used as the input signal of the second. © 2005 Optical Society of America

OCIS codes: 190.4970, 190.7110, 190.4410, 230.4320.


Optical amplifiers with broadband gain are important for a variety of scientific and technological applications. In recent years, optical parametric amplifiers (OPAs) have been used for the amplification of femtosecond pulses, ${ }^{1,2}$ as they present several advantages compared with laser amplifiers.

Quasi phase matching (QPM) allows operation at any desired wavelength with the largest component of the nonlinear susceptibility tensor. ${ }^{3}$ Another important advantage of QPM is the ability to tailor the spectral properties of the nonlinear response through the use of chirped gratings.

Although a single chirped QPM grating can provide a broad parametric gain bandwidth, it does not influence the phase of the amplified signal and as such does not provide the means to compensate for material group-delay dispersion. To address this problem, we propose a tandem-grating OPA that can provide the desired gain and group-delay spectra.
The tandem OPA design is shown in Fig. 1. It makes use of the fact that the signal and idler waves propagate at different velocities inside the crystal. Since the position of the perfect-phase-matching point in a chirped QPM grating varies with frequency, the idler wave experiences a frequencydependent group delay. By use of the idler of the first amplifier as the input signal of the second, it is possible to introduce frequency-dependent group delay to the idler output of the second, whose frequency is the same as the initial signal wave. This controllable group delay can be used to compensate for material group-velocity dispersion. As will be explained in detail below, the desired gain and group-delay spectra can be achieved because we can engineer the position of the perfect-phase-matching point and the local chirp rate at that point simultaneously.

In our tandem OPA design the coupling strength between the signal and idler waves is adiabatically switched on and off at each end of each QPM grating. This tapering alleviates the gain and phase ripple
that would accompany any abrupt switching. Also, to maintain a flat gain over a large bandwidth, we cannot operate in the pump-depletion regime.

Here are some basic results on chirped QPM gratings. The equations describing the spatial evolution of a single frequency component $\omega_{s}$ of the signal and its corresponding idler wave, $\omega_{i}=\omega_{p}-\omega_{s}$, in the presence of an undepleted pump field at frequency $\omega_{0}$ are ${ }^{4}$

$$
\begin{align*}
& \frac{\mathrm{d} A_{s}}{\mathrm{~d} x}=i \gamma(x) A_{i}^{*} \exp [i \phi(x)],  \tag{1}\\
& \frac{\mathrm{d} A_{i}^{*}}{\mathrm{~d} x}=-i \gamma(x) A_{s} \exp [-i \phi(x)] . \tag{2}
\end{align*}
$$

Envelopes $A_{s}$ and $A_{i}$ squared are photon fluxes. The coupling strength is given by $\gamma(x)$ $=\left(\omega_{s} \omega_{i} / n_{s} n_{i}\right)^{1 / 2}\left[d_{\text {eff }}(x) / c\right]\left|E_{0}\right|$, where $n_{s}$ and $n_{i}$ are their refractive indices, $d_{\text {eff }}(x)$ is the effective nonlinear coefficient (which is spatially switched on and off adiabatically to prevent ripple in the amplification spectrum), $c$ is the speed of light in vacuum, and $\left|E_{0}\right|$ is the amplitude of the pump electric field. The wavevector mismatch between the three waves is $\Delta k=\Delta \widetilde{k}$ $-K_{g}$, where $\Delta \widetilde{k}=k_{0}-k_{s}-k_{i}$ is the intrinsic wave-vector


Fig. 1. Schematic representation of the tandem chirped QPM design, showing the tapered coupling strength. $\mathrm{S}_{1,2}$ and $\mathrm{I}_{1,2}$ stand for signal and idler of the first and second gratings, respectively.
mismatch and $K_{g}(x)=2 \pi / \Lambda_{g}(x)$ is the positiondependent wave vector associated with the QPM grating of period $\Lambda_{g}(x)$. This results in an accumulated phase mismatch $\phi(x)=\int_{0}^{x} \Delta k\left(x^{\prime}\right) \mathrm{d} x^{\prime}$, where the input plane of the grating is at $x=0$. We consider gratings where $K_{g}(x)$ and therefore $\Delta k(x)$ are slowly varying, monotonic functions of position. We denote the rate of change of the grating wave vector, or chirp rate, by $\kappa^{\prime}(x)=\mathrm{d} K_{g}(x) / \mathrm{d} x=-\mathrm{d} \Delta k(x) / \mathrm{d} x$.

Optical parametric amplification in a chirped QPM grating, as described by Eqs. (1) and (2) is analogous to three-wave convective parametric instabilities in inhomogeneous plasma, the theory of which was given by Rosenbluth ${ }^{5}$ and used to calculate the single-pass gain of chirped-grating OPAs. ${ }^{6}$ The signal power gain, obtained through a WKB analysis, is given by the Rosenbluth amplification factor ${ }^{5}$ :

$$
\begin{align*}
G\left(\omega_{s}\right) & =\exp \left\{2 \int_{x_{\mathrm{tp} 1}}^{x_{\mathrm{tp} 2}}\left[\gamma^{2}-\left(\kappa^{\prime} / 2\right)^{2}\left(x-x_{\mathrm{pm}}\right)^{2}\right]^{1 / 2} \mathrm{~d} x\right\} \\
& =\exp \left[\frac{2 \pi \gamma^{2}\left(x_{\mathrm{pm}}\right)}{\kappa^{\prime}\left(x_{\mathrm{pm}}\right)}\right] \tag{3}
\end{align*}
$$

In these equations, $x_{\mathrm{pm}}$ is the perfect-phase-matching point, defined by $K_{g}\left(x_{\mathrm{pm}}\right)=\Delta \widetilde{k}\left(\omega_{s}\right)$, and the integration is carried out between the two turning points $x_{\operatorname{tp1,2}}$ $=x_{\mathrm{pm}} \pm 2 \gamma / \kappa^{\prime}$, which are the limits in which we have significant amplification. This expression indicates that the waves stay in phase and interact coherently over a distance given by $4 \gamma / \kappa^{\prime}$ around $x_{\mathrm{pm}}$. Inside the interaction region the gain rate is spatially nonuniform, decreasing from the uniform medium value as one moves to either side of $x_{\mathrm{pm}}$. In contrast, in a uniform grating the waves would grow exponentially with increasing interaction length.

Equation (3) describes the amplification of an infinitely long or tapered-gain grating, and as such does not account for gain ripple caused by the hard edges of a finite-length grating. This ripple originates from the fact that before and after the amplification region the amplitudes of the interacting waves oscillate because of the nearly phase-matched interactions. As such, nearby frequencies have small but different preamplification and postamplification factors, which are superposed on the essentially constant gain provided by the main amplification region, resulting in a rippled spectrum. This ripple can be reduced by adiabatically turning off the coupling strength at the edges of the grating.

Since the length of the amplification region is small compared with the grating length, most of the accumulation of phase occurs before and after the gain region, where the waves propagate at their respective velocities essentially independently of each other. The total group delays at a particular frequency can be approximated by

$$
\begin{equation*}
\tau_{s}\left(\omega_{s}\right)=\frac{L}{v_{s}\left(\omega_{s}\right)} \tag{4}
\end{equation*}
$$

$$
\begin{equation*}
\tau_{i}\left(\omega_{i}\right)=\frac{x_{\mathrm{pm}}\left(\omega_{s}\right)}{v_{s}\left(\omega_{s}\right)}+\frac{L-x_{\mathrm{pm}}\left(\omega_{s}\right)}{v_{i}\left(\omega_{i}\right)} \tag{5}
\end{equation*}
$$

where $v_{s}$ and $v_{i}$ are group velocities of the signal and idler waves, respectively.

A major limitation of amplifiers using a single crystal is the variation in group delay caused by material dispersion. For applications such as ultrashort-pulse amplification it is desirable to minimize the amount of group-delay dispersion introduced by the amplifier. This control can be obtained through the use of a tandem pair of gratings, in which both the gain and the group-delay spectra are engineered, as shown below.

The gain of the tandem OPA is approximately equal to the product of the gains of each grating as given in Eq. (3), provided they are both large. This leads to the following expression for the total gain involving the frequency-dependent perfect-phasematching points in both crystals, $x_{\mathrm{pm}, 1}$ and $x_{\mathrm{pm}, 2}$ :

$$
\begin{align*}
\ln G\left(\omega_{s}\right)= & 2 \pi \gamma^{2}\left(\omega_{s}\right)\left(-\left.\frac{\mathrm{d} \Delta \tilde{k}}{\mathrm{~d} \omega}\right|_{\omega_{s}}\right)^{-1} \\
& \times\left(\left.\frac{\mathrm{d} x_{\mathrm{pm}, 1}}{\mathrm{~d} \omega}\right|_{\omega_{s}}+\left.\frac{\mathrm{d} x_{\mathrm{pm}, 2}}{\mathrm{~d} \omega}\right|_{\omega_{s}}\right) \tag{6}
\end{align*}
$$

The grating functions must be such that the perfect-phase-matching points corresponding to the edges of the bandwidth are situated at the ends of the grating. These requirements allow elimination of the two unknowns contained in Eq. (6) [namely, the ratio $\ln (G) / \gamma^{2}$ and the integration constant]. The total group delay is obtained with Eq. (5):


Fig. 2. (a) Amplification spectrum of the tandem QPM grating OPA. The total gain between 680 and 800 nm is $1.5 \times 10^{7}$, with a root mean deviation error of $9 \%$. Dashed curves, with tapering of the gain coefficient. Thin curves, without tapering, showing the gain ripple. (b) Group delays $\tau_{1,2}$ of each grating.


Fig. 3. (a) Maximum group-delay variation across the amplification bandwidth. (b) Phase-matching period versus position for three tandem pairs, corresponding to amplification bandwidths of 90,175 , and 270 nm , identified by labels (1), (2), and (3) in (a).

$$
\begin{align*}
\tau\left(\omega_{s}\right)= & \frac{x_{\mathrm{pm}, 1}\left(\omega_{s}\right)+\left[L_{2}-x_{\mathrm{pm}, 2}\left(\omega_{s}\right)\right]}{v_{s}\left(\omega_{s}\right)} \\
& +\frac{\left[L_{1}-x_{\mathrm{pm}, 1}\left(\omega_{s}\right)\right]+x_{\mathrm{pm}, 2}\left(\omega_{s}\right)}{v_{i}\left(\omega_{s}\right)} \tag{7}
\end{align*}
$$

where $L_{1}$ and $L_{2}$ are the lengths of each grating. Finally, after $x_{\mathrm{pm}, 1}\left(\omega_{s}\right)$ and $x_{\mathrm{pm}, 2}\left(\omega_{s}\right)$ are obtained, the actual grating profile can be determined with phasematching condition $K_{g}\left[x_{\mathrm{pm}}\left(\omega_{s}\right)\right]=\Delta \widetilde{k}\left(\omega_{s}\right)$.
For example, consider two periodically poled crystals of stoichiometric lithium tantalate, ${ }^{7}$ whose effective nonlinear coefficient ${ }^{8}$ is $2 / \pi \times 15 \mathrm{pm} / \mathrm{V}$. Our pump laser is at 532 nm with an intensity of $1 \mathrm{GW} / \mathrm{cm}^{2}$, and the amplification bandwidth ranges from 680 to 800 nm , corresponding to a transformlimited pulse duration of approximately 10 fs . We determine the grating profiles by solving Eqs. (6) and (7). We are free to choose the grating lengths as long as their ratio is preserved; we choose lengths of 5.0 and 2.2 cm , respectively. Figure 2(a) shows the amplification spectrum of both gratings and their product, obtained from numerical integration of Eqs. (1) and (2) using a fourth-order Runge-Kutta method. The total gain of the amplifier between 680 and 800 nm is $1.5 \times 10^{7}$ with a root mean deviation error of $9 \%$. Without tapering, the gain ripple of each crystal has a magnitude of $6 \%$ and $8 \%$ around the aver-
age value in the center of the passband and up to $20 \%$ and $25 \%$ at the edges, respectively. The ripple can be kept below $0.1 \%$ by tapering the coupling coefficients according to the rule

$$
\begin{equation*}
\frac{\gamma(x)}{\gamma_{\max }}=a+b \times \tanh \left(\frac{x-l_{1}}{w_{1}}\right) \times \tanh \left(\frac{L-x-l_{2}}{w_{2}}\right) \tag{8}
\end{equation*}
$$

where $a$ and $b$ are constants chosen so that $\gamma(0)$ $=\gamma(L)=0$ and with the values $l_{1}=l_{2}=w_{1}=w_{2}=0.02 L$. Figure 2(b) shows the group delay of each grating; their sum is essentially constant over the entire bandwidth.

It is instructive to compare a tandem grating design with a single grating to see to what extent the group-delay dispersion can be compensated when the bandwidth is increased. Figure 3(a) shows the groupdelay variation across the spectrum as a function of bandwidth of a tandem design whose total length is fixed at 5 cm compared with that of a single, linearly chirped QPM grating of the same length. As long as the bandwidth is not larger than 175 nm , it is possible to completely cancel the material dispersion to all orders. For larger values of the bandwidth some group-delay dispersion must be tolerated. Figure 3(b) shows the phase-matching period of each pair of gratings as a function of position for three different bandwidths.

In this Letter we have described an OPA design using tapered coupling strength and tandem chirped QPM gratings to obtain controlled gain and groupdelay spectra. In subsequent papers, additional analysis combined with numerical simulations will give a more detailed description of tapering options and address pulse distortion, transverse beam overlap effects, pump depletion, and self-focusing.

This work was sponsored by the U.S. Air Force Office of Scientific Research under grants F49620-02-10240 and F49620-01-1-0428. M. Charbonneau-Lefort (mathieuc@stanford.edu) acknowledges additional support from the Natural Sciences and Engineering Research Council of Canada. The work of B. Afeyan was supported by U.S. Department of Energy Small Business Innovative Research Phase II grant DE-FG03-01ER83294.

## References

1. A. Dubietis, G. Jonusauskas, and A. Piskarskas, Opt. Commun. 88, 437 (1992).
2. A. Galvanauskas, A. Hariharan, D. Harter, M. A. Arbore, and M. M. Fejer, Opt. Lett. 16, 210 (1998).
3. M. M. Fejer, G. A. Magel, D. H. Jundt, and R. L. Byer, IEEE J. Quantum Electron. 28, 2631 (1992).
4. Y. R. Shen, The Principles of Nonlinear Optics (Wiley, New York, 1984).
5. M. N. Rosenbluth, Phys. Rev. Lett. 29, 565 (1972).
6. K. L. Baker, Appl. Phys. Lett. 82, 3841 (2003).
7. A. Bruner, D. Eger, M. B. Oron, P. Blaus, M. Katz, and S. Ruschin, Opt. Lett. 28, 194 (2003).
8. J.-P. Meyn and M. M. Fejer, Opt. Lett. 22, 1214 (1997).
