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Investigation of mechanical loss factors of some candidate materials for the test masses of gravitational wave detectors

S. Rowan [a], G. Cagnoli [a], P. Sneddon [a], J. Hough [a,*], R. Route [b], E.K. Gustafson [b], M.M. Fejer [b], V. Mitrofanov [c]

[a] *GEO 600 Project, Department of Physics and Astronomy, University of Glasgow, Glasgow, G12 8QQ, UK*
[b] *Stanford University Gravitational Waves Group, Ginzton Laboratory, Stanford University, Stanford CA 94305-4085, USA*
[c] *Department of Physics, Moscow State University, Moscow, Russia*

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Abstract

This Letter describes the results of investigations into the mechanical losses of a selection of crystalline materials with the potential for use as the test masses in advanced gravitational wave detectors. We have measured loss factors of 3.7×10^{-9} for HEM sapphire, to our knowledge the lowest measured loss factors of sapphire grown by a method suitable to produce test masses of the size and quality needed for use in advanced interferometric detectors. We also present the first measurements of the mechanical loss factors of YAG and Spinel at frequencies of interest for gravitational wave detection.

1. Introduction

For all the long baseline gravitational wave detectors under construction, and being planned, the operating frequency range lies between the lowest internal resonances of the test masses and the resonances of their pendulum suspensions. It is expected that the most significant noise source at the lower end of the operating frequency range will be thermal noise in the tails of the relevant resonances. The narrower the resonances, the lower is the off-resonance thermal noise; thus the reduction of resonance widths or the increase in their quality factors is an experimental challenge of great importance. In this Letter measurements of the quality factor of a number of materials important for, or potentially important for, the development of gravitational wave detectors are presented and discussed.

2. Thermal noise

For any simple harmonic oscillator such as a mass hung as a pendulum, the spectral density of thermal motion of the mass at an angular frequency ω, can be expressed as [1]

$$\tilde{x}^2(\omega) = \frac{4k_B T w_0^2 \phi(\omega)}{\omega m\left[\left(\omega_0^2-\omega^2\right)^2+\omega_0^4\phi^2(\omega)\right]}, \qquad (1)$$

where k_B is Boltzmann's constant, T is the temperature of the mass, m, and $\phi(\omega)$ is the loss factor of

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[*] Corresponding author. E-mail: j.hough@physics.gla.ac.uk

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the oscillator. This loss factor at the resonance angular frequency, ω_0, is the inverse of the quality factor, Q, of the resonance.

For a pendulum, most of the energy associated with the pendulum motion is stored as lossless gravitational potential energy, with just some small fraction of the energy stored in the bending of the pendulum suspension fibres. The loss factor of the pendulum resonance is thus lower than that of the material of the wires or fibres that suspend the mass. This dilution of loss does not happen for the internal resonances of the test mass, however, and from Eq. (1) it can be seen that the contribution to thermal noise of a test mass below it's first resonance frequency and resulting from that resonance has the form

$$\tilde{x}^2(\omega) \simeq \frac{4k_B T\phi(\omega)}{m\omega\omega_0^2}. \tag{2}$$

In general, for most bulk materials it appears that the intrinsic material loss factor is essentially independent of frequency over the range of interest for gravitational wave detectors [2–4] and thus the thermal motion of the front face of a test mass increases as $1/\sqrt{\omega}$ towards lower frequencies. Analysis of typical detector designs quickly shows that the absence of loss dilution is very important and that for silica masses suspended on silica fibers or ribbons the pendulum thermal noise need only become important at frequencies around 10 Hz and below ie at frequencies close to the natural low frequency limit set by gravity gradient effects [5–8]. Thus it is the internal thermal noise which is the most significant limitation and which must be reduced. In fact, Eq. (2) above is optimistic as there are also contributions of thermal noise from the higher frequency resonant modes of the mass to be taken into account. This has been carried out in a careful analysis by Gillespie and Raab [9] and they have shown that the thermal noise from the internal modes can be expressed in the form

$$\tilde{x}^2(\omega) = \beta\frac{4k_B T\phi_{\mathrm{mat}}(\omega)}{m\omega\omega_0^2}, \tag{3}$$

where m is the mass of the test mass, ω_0 is the resonant angular frequency of the fundamental mode, $\phi_{\mathrm{mat}}(\omega)$ is the intrinsic material loss, and β is a correction factor to include the effect of summation of the motion over the higher order modes of the test mass. This factor takes into account the effect of the size of the laser beam interrogating the front face of the test mass and the effect of the different effective masses of the higher order modes. Typically β is a number less than 10. This summation further emphasises the need to use materials of low loss factor for the test masses of the gravitational wave detectors being developed. To achieve the sensitivities desired for advanced gravitational wave detectors of $h \simeq 10^{-22}/\sqrt{Hz}$ at 10 Hz, test masses with mechanical quality factors of $\geq 10^8$ are required. It should be noted that a different and more general treatment of internal thermal noise using evaluation of the relevant mechanical impedance has been carried out by Bondu et al [10]. This was based on work of Yuri Levin [11] and gives good agreement with the results of Gillespie and Raab.

3. Aspects of test mass materials for long baseline gravitational wave detectors

There are a number of constraints which affect the selection of material for the test masses of long baseline gravitational wave detectors. The material should:

1. have a very low loss factor at room temperature as discussed above;
2. have high thermal conductivity and low thermal expansion in order to minimise mechanical distortion when heat is deposited in it by the laser beam illuminating the interferometer [12];
3. have very low optical loss at the wavelength of the laser light to minimise heat deposited [12];
4. have if possible a low value of change of refractive index with temperature to minimise thermal lensing effects for transmitted laser beams [12];
5. be suitable for polishing and coating to sub angstrom surface roughness;
6. be capable of being produced in suitable size (up to mass of several tens of kg to reduce photon recoil effect and indeed the effects of the Heisenberg Uncertainty Principle);
7. have oxidised aluminium or silicon in their makeup so that they chemically react with the alkali metal hydroxides and thus can be bonded to

suspension elements by the low loss hydroxy-catalysis bonding technique [13,14]

If reflective optics are used for the interferometer [15], the optical absorption of the test mass material is no longer important, but the availabilty of non-transmissive components of suitable performance seems to be some years away at present.

There are few materials which have a low enough mechanical loss at room temperature to be of use in gravitational wave detectors, and these tend to be very pure glasses such as fused quartz or fused silica or single-crystals materials such as sapphire and silicon. Experiments in a number of laboratories [2,3,9,16] are suggesting that the losses in fused silica (synthetic in origin) range between 2×10^{-7} and 3×10^{-8} and are lower than those in fused quartz (natural in origin) by a significant factor. Thus the material adopted for the test masses in LIGO [17], VIRGO [18], GEO 600 [19] and TAMA 300 [20] is fused silica manufactured either by Heraeus or Corning. Silica is also attractive because of potentially high optical quality, size of pieces available, ease of polishing and coating, and low optical absorption loss [21] in some types such as Suprasil SV from Heraeus. The mechanical loss factor of fused silica will however set a limit to the thermal noise performance achieveable with current detectors, and thus to improve performance future detectors are likely to need alternative test mass materials. A material whose mechanical properties have been studied for a number of years is sapphire (Al_2O_3). Very low mechanical loss factors, down to 3×10^{-9}, have been reported in Russia, for material grown using the horizontal oriented crystallization process. [22] However pieces of the size and optical quality required for gravitational wave detectors must be made by the Heat Exchanger Method (HEM) process [23] and measurements in Australia [24] of the mechanical losses of material made in this way have yielded loss factors of 2×10^{-8} a value significantly poorer than that for the horizontal oriented crystallization material tested in Russia and not significantly better than that for the best fused silica. Thus it clearly was important to investigate this further, and work in this area will be described in the next section.

Sapphire made by the HEM process is grown most easily along the *a* and *m* crystallographic directions and thus the largest diameter cylindrical pieces available have their axes in these directions; the crystal structure of sapphire results in anisotropy of its mechanical properties along the two axes at right angles to the growth axis of these boules. Both the inhomogeneity and birefringence of the uniaxial crystal may lead to constraints on the use of such cylinders as test masses for gravitational wave detectors [25]. If these effects prove insurmountable the use of pieces of sapphire cored out along the *c* axis should get around these problems and the development of pieces of adequate size is currently under investigation for the LIGO project. Measurements of mechanical loss at very high (GHz) frequencies have shown that certain other crystals sych as YAG (yttrium aluminium garnet) and spinel (magnesium aluminium oxide) can have loss factors of the same order, or even lower than sapphire [26]. These crystals are broadly similar to sapphire in mechanical properties, but more symmetrical in crystal structure and have the advantage over sapphire of being optically isotropic thus removing any potential complexity associated with using a material with inherent birefringence as a transmissive optic. It was thus of interest to have a measure of the mechanical loss of these materials at frequencies closer to those of interest for gravitational wave detection and in the experimental work to be described here preliminary measurements of the mechanical losses of two other materials – YAG (yttrium aluminium garnet) and spinel (magnesium aluminium oxide) – were made.

4. Measurement of mechanical losses in materials

4.1. Experimental philosophy

In order to fully characterise the mechanical loss of a material for use for the test masses of a gravitational wave detector, it is desirable to directly measure this quantity over the range of operating frequencies of the detector. Unfortunately, this is difficult to carry out experimentally. A system measuring the anelastic relaxation of the material has been devised for this purpose in Syracuse [27] but as yet it does not have the sensitivity required to characterise very low loss materials such as sapphire. Thus, it is assumed that mechanical loss is independent of frequency, as has been shown experimentally for fused quartz and fused silica, and that the loss factors associated with the resonant modes of specimens of the material will be a good indicator of performance. In practice a suspension or support for the test mass has to be carefully chosen so as to avoid the introduction of excess mechanical loss in the measurement. A single loop of fibre or wire is a good candidate for this, in particular for the case of the fundamental mode of a cylinder since for isotropic materials, the wire loop lies at the node of the displacement of the mode along the cylinder axis and the highest room temperature Q measurements in materials have been obtained using this method [28]. There is however some movement in the radial direction due to the Poisson's ratio of the material; this can still lead to loss by frictional effects at the mass surface and by exciting the resonant modes of the suspending loop. The material loss factors measured and discussed in this Letter are for the fundamental modes of cylinders of the different materials.

4.2. Experimental technique

The measurement is based on the fact that the amplitude A of the freely decaying resonant motion of an excited harmonic oscillator decays as

$$A = A_0 e^{-\omega_0 t/(2Q)}, \tag{4}$$

where A_0 is the initial amplitude of the motion, Q is the quality factor of the resonance and $\phi(\omega_0) = 1/Q$. Thus if a resonant mode of a specimen is excited and the amplitude is recorded as a function of time the quality factor and loss factor of the material at that mode frequency can be measured.

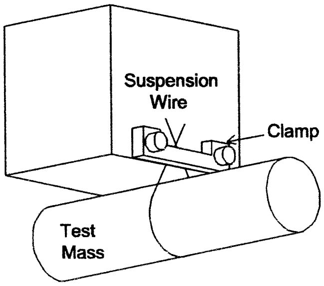

Fig. 1. Suspension arrangement used to support cylindrical samples of materials for Q measurements.

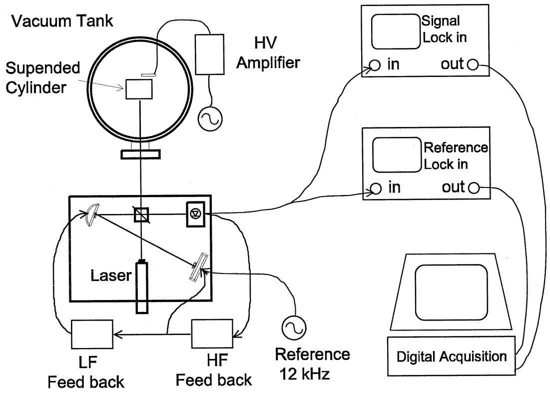

Fig. 2. Arrangment for sensing the motion of test mass.

Specimens to be tested were cylindrical in shape and were hung by a loop of silk fibre or wire from a steel clamp, as illustrated in Fig. 1, which was mounted on an aluminium plate supported by short aluminium legs inside a vacuum tank, which can be pumped down to below 10^{-5} mb.

The arrangement was similar to that used in earlier work [29,13] except that the length of the suspension fibres was kept as small as possible. This minimises the chance of the resonant frequency of the specimen overlapping with a resonance of the suspension fibres as this has been demonstrated both theoretically and experimentally to result in artificially low values of measured Q [13,28,29]. The fundamental resonant mode of each specimen was excited electrostatically and the motion of the front face detected interferometrically. A 1 cm diameter disc of aluminium was evaporated onto the front face of each sample to form a mirror which formed the end of one arm of a Michelson interferometer. The beamsplitter and second arm of the interferometer were external to the tank. Two mirrors were positioned in a folded arrangement (see Fig. 2) to ensure that the second arm of the interferometer was of approximately the same length as the first.

One of these mirrors was mounted on a loudspeaker and one on a PZT transducer. This interferometer was illuminated with light from a Helium–Neon laser.

A fraction of the detected signal at the interferometer output was, after suitable amplification and filtering, fed back to the loudspeaker and pzt, locking

Table 1
Results of Q measurements of various materials of interest for use as the test masses in interferometric gravitational wave detectors.

Material	Dimension diameter × length	Frequency	Q	Suspension fibers
YAG	25 mm × 101 mm	38.930 kHz	$(2.9 \pm 0.4) \times 10^7$	silk
Spinel	19 mm × 76 mm	66.196 kHz	$(0.8 \pm 0.4) \times 10^7$	silk
HEMEX sapphire	30 mm × 100 mm	53.591 kHz	$(13.4 \pm 0.4) \times 10^7$	silk
HEMEX sapphire	30 mm × 100 mm	53.591 kHz	$(25.9 \pm 0.5) \times 10^7$	tungsten

the interferometer to the low frequency motions of the test mass. The bandwidth of the locking system was chosen to allow the much smaller signals resulting from the vibrations of the mass at its internal resonant mode frequencies to be detected in a linear way at the error point of the system.

Signals at the fundamental mode frequency of the test mass were amplified and passed through a band-pass filter and sent to the signal input of a lock-in amplifier (Stanford Research Systems Model SR 830 DSP). The internal frequency of the lock-in amplifier was set to be approximately 10 Hz away from the test mass frequency. The output signal from the lock-in amplifier was a signal at the beat frequency of 10 Hz, the amplitude of which decayed at a rate identical to that of the much higher frequency test mass signal. The amplitude of this beat signal was recorded using a computer data acquisition system, and from measurements of the recorded amplitude decay, the Q factor of a test mass mode could thus be calculated using Eq. (4). In these measurements the measured signal was significantly larger than the background noise in the measurement system and so no correction was applied to the measured amplitude [14].

In order to monitor the effects of potential gain changes in the system, which can result from drift of the locking point in the interferometer, a signal at 12 kHz from a stable generator was imposed on the piezo-electric transducer and was detected and displayed in the same way as the main signal. Fluctuations in the size of this signal were used to correct the amplitude of the main signal during the ring-down measurements.

Specimens were hung either with fine silk thread (120 microns diameter) or with polished tungsten wire of diameter 50 microns. In each case a fine coating of animal fat was applied to the mass by greasing a slightly heavier silk fibre and initially hanging the mass with that. The mass was then let back down and the heavier fibre replaced by a lighter one or by tungsten wire. To a large extent the method adopted emulates the method used in Moscow for obtaining the Q values of specimens of sapphire made by the technique of horizontal oriented crystallization.

4.3. Experimental results

In Table 1 the materials, their dimensions, their method of suspension and measured Q values are summarised.

As can be seen the Q of the HEMEX sapphire from Crystal Systems is close to the best values reported in the Russian measurements and significantly better than earlier measurements of this material [24]. However, the limiting part played by the suspensions can clearly be seen in that the best values were only obtained with the polished tungsten suspension fibre. A ringdown measurement for the mass suspended on tungsten is shown in Fig. 3. The measured values with the silk suspension were noti-

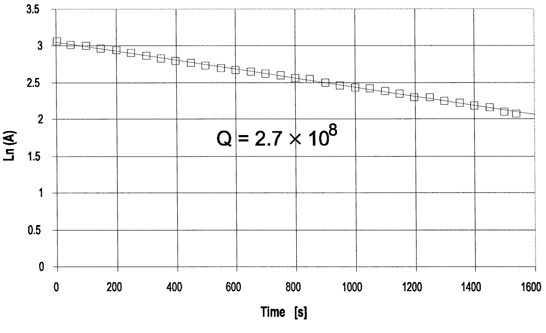

Fig. 3. Logarithmic decay of the amplitude of the fundamental resonant mode of the HEMEX Sapphire.

cably poorer. The Q factors for both the YAG and the Spinel are lower than that for sapphire but are still encouraging in that they are in the range found for high quality fused silica.

One noticable fact however was that the surface qualities of the YAG and Spinel samples appeared different from that of the sapphire or of the samples of fused silica normally tested. For example measurements with an atomic force microscope suggested that roughness of the sapphire over a scale of approximately 40 microns was around 10 nm rms over a spatial bandwidth of approximately 6 cycles per micron, while the roughness of the YAG sample was close to two times greater with the addition of many scratches of 80–90 nm depth separated by around 2 microns. The surface of the spinel was broadly similar to that of the YAG but also had many pits and surface tearouts, approximately 20 nm deep by 20 microns wide, seemingly due to material flaws that gave way during polishing. Thus it is possible that if the surface quality of these samples is improved their Q factors may also improve, a subject for future investigation.

5. Conclusions

It has been shown that commercially available Hemex sapphire grown by the Heat Exchanger Method has a Q or loss factor equivalent to that the Russian samples produced by horizontal oriented crystallization growth, this loss factor being of the level required to make this material useable as the test masses in the advanced stages of interferometric gravitational wave detectors. Currently the optical loss of sapphire is being studied and results are promising [30,31]. Further, the first measurements of the loss factors of both YAG and spinel presented here are low enough to warrant investigation of improvements with further polishing, if large enough pieces can be grown and if optical loss factors of an acceptable value can be achieved.

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References

[1] P.R. Saulson, Phys Rev. D 42 (1990) 2437.

[2] S. Rowan, R. Hutchins, A. McLaren, N.A. Robertson, S.M. Twyford, J. Hough, Phys. Lett. A 227 (1997) 153.

[3] W.J. Startin, M.A. Beilby, P.R. Saulson, Rev. Sci. Inst. 69 (1998) 3681.

[4] A.S. Nowick, B.S. Berry, Anelastic Relaxation in Crystalline Solids, Academic Press.

[5] R.E. Spero, in: M.M. Nieto et al. (Eds.), Science Underground, AIP, New York, 1983.

[6] P.R. Saulson, Phys Rev. D 30 (1984) 732.

[7] M. Beccaria et al., Class. Quantum Grav. 15 (1998) 1.

[8] S. Hughes, K.S. Thorne, Phys. Rev. D 57 (1998) 122002.

[9] A. Gillespie, Thermal Noise in the Initial LIGO Interferometers, PhD Thesis, California Institute of Technology, 1995.

[10] F. Bondu, P. Hello, J-Y. Vinet, Phys. Lett. A 246 (1998) 227.

[11] Yu. Levin, Phys. Rev. D 42 (1998) 659.

[12] W. Winkler, K. Danzmann, A. Ruediger, R. Schilling, Phys. Rev. A 44 (1991) 7022.

[13] S. Rowan, S.M. Twyford, J. Hough, D.-H. Gwo, R. Route, Phys. Lett. A 246 (1998) 471.

[14] S.M. Twyford, PhD thesis University of Glasgow, 1999.

[15] K.-X. Sun, R.L. Byer, Opt. Lett. 23 (1997) 567.

[16] B.S. Lunin, S.N. Torbin, M.N. Danchevskaya, I.V. Batov, Moscow Univ. Chem. Bull. Ser. 2 (1994) 24.

[17] B.C. Barish, LIGO: Status and Prospects, in: K. Tsubono, M.K. Fujimoto, K. Kuroda (Eds.), Universal Academy Press, Inc. Tokyo, 1997, pp. 155.

[18] A. Brillet et al., VIRGO Status Report, November 1996, in: K. Tsubono, M.K. Fujimoto, K. Kuroda (Eds.), Gravitational Wave Detecton, Universal Academy Press, Inc. Tokyo, 1997, 163.

[19] J. Hough et al., GEO 600: Current Status and some Aspects of the Design, in: K. Tsubono, M.K. Fujimoto, K. Kuroda (Eds.), Gravitational Wave Detecton. Universal Academy Press, Inc. Tokyo, 1997, 175,

[20] K. Tsubono and the TAMA Collaboration, Tama Project, in: K. Tsubono, M.K. Fujimoto, K. Kurodo (Eds.), Gravitational Wave Detecton, Universal Academy Press, Inc. Tokyo, 1997, 183.

[21] C. Boccara V, Loriette VIRGO note (1993) VIR-NOT-PCI-1380-65.

[22] V.P. Mitrofanov, V.S. Shiyan, Kristallografiya 24 (1979) 307.

[23] http://www.crysys.com/hem.html.

[24] M. Taniwaki, Design and Verification of low acoustic loss suspension systems for interferometic gravitational wave detectors PhD thesis University of Western Australia, 1999.

[25] L. Ju, M. Notcutt, D. Blair, F. Bondu, C.N. Zhao, Phys. Lett. A 218 (1996) 3.
[26] B. Auld, Acoustic Fields and Waves in Solids, vol. II, Wiley, 1993.
[27] M. Beilby, P.R. Saulson, A. Abramovici, Rev. Sci. Instrum. 69 (1996) 2539.
[28] V.B. Braginsky, V.P. Mitrofanov, V.I. Panov, Systems with Small Dissipation University of Chicago Press, Chicago, 1985.
[29] J.E. Logan, N.A. Robertson, J. Hough, Phys. Lett. A 170 (1992) 352.
[30] Proc. 3rd Edoardo Amaldi Conference on Gravitational Waves - Pasadena CA.
[31] Private communication.