Fiber-laser frequency combs with subhertz relative linewidths

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We investigate the comb linewidths of self-referenced, fiber-laser-based frequency combs by measuring the heterodyne beat signal between two independent frequency combs that are phase locked to a common cw optical reference. We demonstrate that the optical comb lines can exhibit instrument-limited, subhertz relative linewidths across the comb spectra from 1200 to 1720 nm with a residual integrated optical phase jitter of ~1 rad in a 60 mHz to 500 kHz bandwidth. The projected relative pulse timing jitter is ~1 fs. This performance approaches that of Ti:sapphire frequency combs. © 2006 Optical Society of America

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The output of a femtosecond fiber laser consists of a train of optical pulses at a repetition frequency f_r. In the frequency domain, this output is a comb of lines at frequencies $f_n = nf_r + f_{\text{ceo}}$, where n is the mode number and f_{ceo} is the carrier-envelope offset frequency. Following the original work with femtosecond Ti:sapphire lasers,[1,2] fiber-laser combs have been stabilized by phase-locking both f_r and f_{ceo} to a microwave reference, yielding a valuable tool for frequency metrology.[3–6] Fiber lasers have a number of advantages over their Ti:sapphire cousins in that they are more compact, are capable of turnkey, long-term operation, are less expensive with lower power consumption, are compatible with existing fiber optics, and cover the telecommunication window. However, while Ti:sapphire combs have exhibited subhertz linewidths,[7] fiber-laser frequency comb linewidths have until recently been at the kilohertz level or higher. There is no fundamental reason for these large linewidths,[8] and two combs with low-phase-noise f_{ceo} beat notes have been described.[9,10] Here we demonstrate that f_r can be similarly narrowed by phase locking one optical comb tooth to a cw optical reference. We find that the comb lines can exhibit low residual phase noise and subhertz relative linewidths across the comb. A narrow-linewidth fiber-laser frequency comb should find important applications in metrology for low-phase-noise cw lasers, high-precision optical spectroscopy, coherent Lidar,[11] fiber transport of frequency standards,[12,13] and optical clocks.

Following the work on Ti:sapphire combs by Bartels *et al.*,[7] we compare two distinct fiber laser frequency combs by phase locking a tooth of each comb to a common, narrow cw reference laser while simultaneously phase locking f_{ceo} by use of the standard f-to-$2f$ technique. Although the details are complicated,[14] essentially the lock to the reference laser fixes a central comb tooth by stabilizing the cavity length while the offset frequency lock removes any breathing motion of the comb about that central, fixed tooth. In our experiments, two very different frequency combs are phase-locked with an 8 MHz difference in their offset frequencies and a fixed integer relationship between their repetition rates. The comb outputs are combined, optically filtered, and detected to generate an 8 MHz heterodyne beat from different

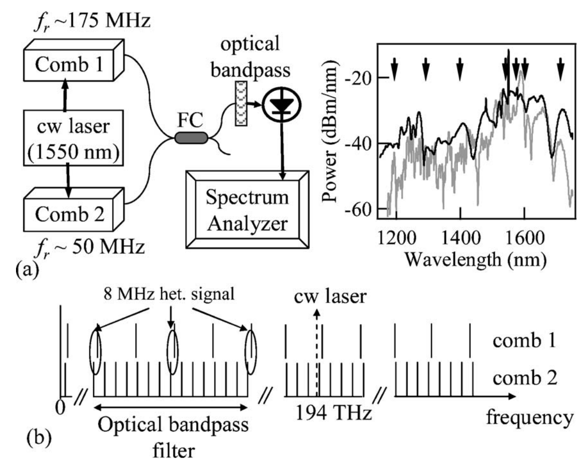

Fig. 1. (a) Schematic of the experimental setup. FC, fiber coupler. Inset, spectra of comb 1 (gray) and comb 2 (black). Arrows indicate the location of the different optical bandpass filter settings for the heterodyne measurements. (b) Comb 1 (top) and comb 2 (bottom), along with the cw reference laser at 1550 nm (194 THz). The circled pairs contribute to the beat note at 8 MHz. (The number of contributing pairs is ~400–4000, depending on the filter bandwidth.)

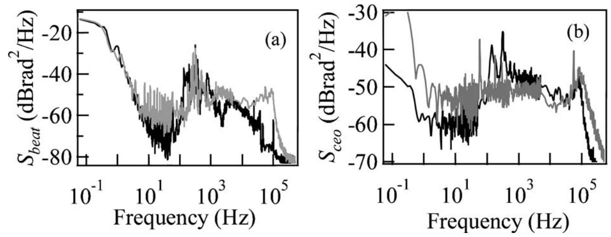

Fig. 2. (a) In-loop phase noise spectral densities for the lock between the cw reference laser and comb 1 (gray) and comb 2 (black). The integrated phase noise of 0.94 and 0.35 rad, respectively, is dominated by the servo bumps at high Fourier frequencies. Extrapolation to the Nyquist frequencies of 92.5 and 25 MHz for the two combs gives integrated phase noises of 0.53 and 1.21 rad. (b) In-loop phase noise for f_{ceo} for comb 1 (gray) and comb 2 (black). The integrated phase noises are 1.5 and 1.0 rad, respectively.

spectral regions across the comb; the beat signal is analyzed to give the residual linewidth and phase noise. We obtain a coherent δ-function peak on the heterodyne signal across the spectrum with a signal-to-noise ratio of ~ 50 dB/Hz and a correspondingly low integrated residual phase jitter of ~ 1 rad.

Figure 1 shows the experimental setup. Comb 1 is based on a Fabry–Perot-type Er-soft-glass-fiber oscillator.[15] Self-starting mode-locking operation is stabilized by a saturable absorber mirror. The oscillator intracavity dispersion is compensated by a chirped fiber Bragg grating to near zero. The oscillator emits a pulse train with a repetition rate $f_{r,1} \sim 175$ MHz and an optical bandwidth of ~ 60 nm. Comb 2 is based on a stretched-pulse Er-fiber ring laser with a repetition rate of $f_{r,2} \sim 50$ MHz and optical bandwidth of 80 nm.[10,16] The combs were on separate optical tables. Each laser emits between 5 and 10 mW, which is amplified in a bidirectional (comb 1) or backward (comb 2) pumped Er-fiber amplifier. The amplified output is spectrally broadened to an octave in UV-enhanced highly nonlinear fiber.[17]

Each comb is stabilized to a common, 1550 nm cw fiber laser by beating an individual, ~ 1 nW comb line against the ~ 1 mW cw laser. The cw fiber laser linewidth was specified at ~ 2 kHz with slow drifts of megahertz over minutes. The two beat signals are detected at $f_{\text{beat},1} = 58$ MHz and $f_{\text{beat},2} = 66$ MHz, respectively, upshifted with a common 1 GHz synthesizer, divided by 64, and phase locked to two separate frequency synthesizers by feeding back to the cavity length with both a slow and a fast piezoelectric transducer (PZT). For comb 1, the two PZTs change the saturable absorber mirror position with a feedback bandwidth limited by resonances at 1 Hz and 70 kHz, respectively. For comb 2, the two PZTs change the fiber length with a feedback bandwidth limited by resonances at 5 and 45 kHz feedback, respectively. The resulting in-loop phase noise spectra are shown in Fig. 2(a), and the corresponding phase-locked coherent peak is shown in Fig. 3. The low $1/f$ shoulder in the phase noise at Fourier frequencies <5 Hz is a result of phase noise on the frequency synthesizers used in the optical phase lock. Despite the very different design of the two fiber laser combs, the performance is very similar. To stabilize the remaining degree of freedom, the offset frequencies of the two combs are phase locked to $f_{\text{ceo},1} = 120$ MHz and $f_{\text{ceo},2} = 112$ MHz through feedback to the pump power.[9,10] In each case, this phase lock results in a strong coherent f_{ceo} peak with a signal-to-noise ratio of 50–57 dB/Hz and a low phase noise [see Fig. 2(b)].

While Fig. 2 gives the residual, in-loop phase noise of the two phase-locked signals, we are interested in how this phase coherence is preserved across the comb. To do this we set the combs' repetition rates to an integer ratio and compare coincident comb teeth (offset by 8 MHz) across the spectrum. The repetition rate depends on both f_{ceo} and f_{beat} as $f_{r,i} = (f_{1550} \pm f_{\text{beat},i} - f_{\text{ceo},i})/n_{1550,i}$, where $f_{1550} = c/1550$ nm is the frequency of the cw laser and $n_{1550,i}$ is an integer identifying the nth mode nearest to f_{1550} of the ith comb.[7] Since both the f_{beat} and the f_{ceo} signals of the two combs differ by 8 MHz, with the appropriate sign choice the repetition rates of the two combs will have a fixed integer relationship. For the cavity lengths of the two combs, we can fix the repetition rates such that $2f_{r,1} = 7f_{r,2}$. Adjusting the repetition-rate phase between the two combs[7] ensures that every second pulse of comb 1 and every seventh pulse of comb 2 arrive coincidentally at the photodetector, generating an 8 MHz heterodyne beat signal that is subsequently recorded with a digital fast Fourier transform instrument. This beat signal represents the average relative linewidth of the collection of comb lines transmitted by the optical filter (see Fig. 1).

Figure 3 demonstrates a strong coherent peak in the heterodyne beat between the two combs at wavelengths of 1200 ± 5, 1300 ± 5, 1400 ± 5, 1540 ± 0.5, 1580 ± 5, 1600 ± 0.5, 1720 ± 15 nm (see Fig. 1), where $\pm x$ indicates the 3 dB bandwidth of the optical filter. This coherent peak approaches a δ-function with a 3 dB linewidth limited by either the spectrum analyzer or uncompensated drifts in any out-of-loop fibers. In this case our coherent peak is instrument limited to 300 mHz over the 48 s acquisition time. The inset shows the heterodyne beat at 1720 nm for a wider 1 MHz span, showing the coherent peak con-

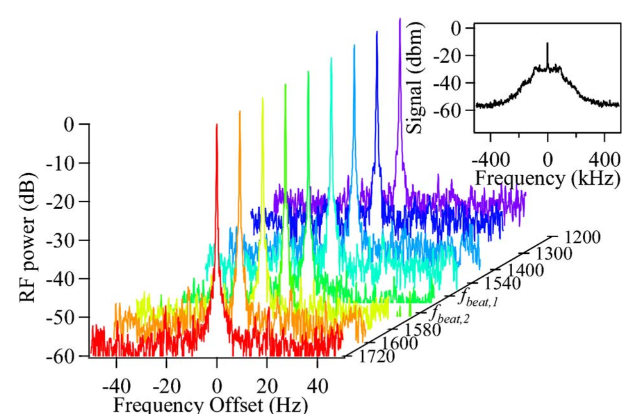

Fig. 3. (Color online) Spectra of the coherent peak across the comb and on $f_{\text{beat},i}$ between the combs and the cw reference laser (resolution bandwidth (RBW) of 0.3 Hz, acquisition time 48 s). Inset, rf spectra on a 1 MHz span and log scale for the heterodyne beat at 1720 nm (RBW = 3 kHz).

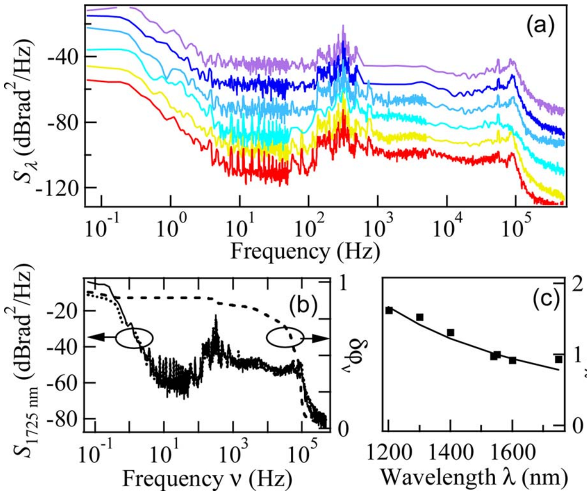

Fig. 4. (Color online) (a) Phase noise PSDs at $\lambda = 1200, 1300, 1400, 1540, 1580, 1720$ nm (top to bottom). The PSDs above 1200 nm are offset by 10 dB from each other for viewing clarity. (b) Comparison of the measured (solid curve) and predicted (dotted curve) phase noise PSD at 1720 nm, which are basically indistinguishable. Also shown is the integrated phase noise, $\delta\varphi_\nu$ (dashed curve, right axis), which reaches 0.9 rad. Extending the white-phase-noise floor to 25 MHz yields a residual phase jitter of 1.03 rad. (c) Measured (squares) and predicted (curve) integrated phase jitter, $\delta\varphi_\lambda$, versus wavelength.

taining 55% of the total power atop a broad noise pedestal. This broad noise pedestal is seen on the heterodyne beat at all wavelengths and is a direct result of phase noise. Indeed, for such a coherent lock, the most physically meaningful measure of the coherence is the phase noise power spectral density (PSD).

Figure 4 gives the phase noise PSDs, $S_\lambda(\nu)$, as function of Fourier frequency, ν. The $S_\lambda(\nu)$ are scaled copies of each other and are limited by the quality of the optical phase locks, as is shown below. (Excess phase noise from supercontinuum generation may exist as a white-noise floor at large ν but was not explored here). The frequency of the nth tooth is $f_n = nf_r + f_{ceo} = r(f_{1550} + f_{beat}) + (1-r)f_{ceo}$, where $r \equiv n/n_{1550}$ and ranges from 0.9 to 1.3 for λ from 1200 to 1720 nm. Therefore, its residual phase noise PSD is $S_n(\nu) = r^2 S_{beat}(\nu) + (1-r)^2 S_{ceo}(\nu) \approx r^2 S_{beat}(\nu)$, since S_{ceo} is not much larger than S_{beat} and we assume negligible correlations between S_{beat} and S_{ceo}. The residual phase noise between the combs is then $S_\lambda \approx (f_\lambda / f_{1550})^2 S_{beat,\Sigma} = (1550/\lambda)^2 S_{beat,\Sigma}$, where $f_\lambda \equiv c/\lambda$, $S_{beat,\Sigma} \equiv S_{beat,1} + S_{beat,2}$, converting from n to λ. (Correlated phase noise on the two combs, most notably from frequency fluctuations of the free-running cw reference laser, will not appear on S_λ) In this simple approximation the optical lock stabilizes the relative repetition rates to $S_{beat,i}/f_{1550}^2$; multiplying this repetition rate noise up to the appropriate optical frequency yields S_λ. Figure 4(b) shows that the measured and predicted S_λ agree well, verifying the simple approximations. The integrated phase jitter, $\delta\varphi_\lambda^2 = \int_{0.06\ \mathrm{Hz}}^{0.5\ \mathrm{MHz}} S_\lambda(\nu) d\nu \propto \lambda^{-2}$ is shown in Fig. 4(c). Finally, returning to the rf heterodyne beat, the fractional power in the coherent peak will be $\exp(-\delta\varphi_\lambda^2)$.[18]

The tight phase lock between the two combs implies a low residual timing jitter. A reasonable upper limit to this jitter on the pulse train of comb 2 is $(2\pi f_{1550})^{-1}[\int_{0.06\ \mathrm{Hz}}^{0.5\ \mathrm{MHz}} (S_{beat,2}(\nu) + S_{ceo,2}(\nu)) d\nu]^{1/2} = 0.9$ fs. Extrapolating to the Nyquist frequency of 25 MHz from the measured white-phase-noise floor increases the jitter to 1.12 fs. Of course, such a low timing jitter remains to be demonstrated. Finally, the low phase noise also translates into excellent relative frequency stability; with a lower-noise frequency synthesizer used in the optical phase lock for comb 2, the counted f_{beat} and f_{ceo} both exhibit a counter-limited standard deviation of <1 mHz at a 1 s gate time, giving a statistical error of 1 mHz/(c/1550 nm) ~ 5×10^{-18} at 1 s, which can certainly support the next generation of optical clocks.

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